A stochastic model for sound propagation over irregular surfaces

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Outline

Objectives, context and proposed approach

Description of the methodology

Construction of the stochastic model
  Probability distribution of $H$, $W$, and $D$
  Description of the reference model
  Description of the stochastic model
  Probabilistic model of $\Gamma$, $\Lambda$ and $\Theta$
  Identifications of the distribution functions parameters

Application and validation

Conclusions and perspectives
Objectives

Development of a computational model for the study of wave propagation over irregular surfaces.
Research context

- Two-dimensional simulations in a homogeneous atmosphere
- Scatterers on the surface are of rectangular shape; they are parametrized by:
  - their height $h$,
  - their width $w$,
  - the spacing between two consecutive objects $d$.
- Scatterers surfaces and backing surface are perfectly rigid
- Backing surface is infinite
Proposed approach

Conventional approach: “exact” or explicit models (FDTD, BEM, ray tracing, etc.)

- Model approximations (e.g., diffraction theory)
- Uncertainties on the system parameters (e.g., scatterers geometry)
- High computational effort
- Relevance of calculated solution (results for one specific case)

Proposed approach: stochastic propagation model

- Very simple mean propagation model
- Associated with a probabilistic approach of uncertainties.
## Proposed approach

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Proposed approach

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- ... associated with a probabilistic approach of uncertainties.
Proposed approach (cont’d 1)

Explicit or “exact” wave propagation model
Proposed approach (cont’d 1)

Explicit or “exact” wave propagation model

Stochastic wave propagation model

Source

Receiver

\[ P_r(\omega) \]

\( (\Gamma, \Lambda, \Theta) \)

Stochastic surface
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Probability distribution of $H$, $W$, and $D$

Information Theory and maximum entropy principle: maximize the uncertainty of the system ("worst-case" scenario)

- Algebraic properties: real, positive quantities
- No dependence between $h$, $w$, and $d$ is assumed

The maximum entropy principle yields

- Independence of random variables $H$, $W$, and $D$
- Probability distributions of $H$, $W$, and $D$ are gamma distributions
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# Probability distribution of $H$, $W$, and $D$

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### Construction of the computational model

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1. **Construction of the probability distributions of the scatterers geometrical parameters** (height $h$, width $w$, spacing $d$ of scatterers)
2. **Obtain reference solutions**
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4. **Identify parameters of $p_\Gamma(\gamma)$, $p_\Lambda(\lambda)$ and $p_\Theta(\theta)$**
5. **Solve the stochastic equations with the Monte-Carlo method**
Reference model

Boundary Element Method (BEM)

- 2D simulations on \([0 - 10] \text{ kHz}\)
- Perfectly reflecting surfaces (horizontal and vertical surfaces)
- The quantity under interest is

\[
L^{\exp}(\omega) = 10 \log_{10} \left( \frac{\left| P^{\exp}(\omega) \right|^2}{\left| p^{\text{free}}(\omega) \right|^2} \right)
\]

- 500 realizations: \(10^{-3}\) convergence on the 1\(^{\text{st}}\) and 2\(^{\text{nd}}\) moment of \(L^{\exp}(\omega)\)
Example:

- \((m_H, \sigma_H) = (40, 8) \text{ cm}\)
- \((m_W, \sigma_W) = (40, 8) \text{ cm}\)
- \((m_D, \sigma_D) = (30, 6) \text{ cm}\)

Figure: Reference model (500 outputs, thin black lines) and the mean value estimate \(m_{L^{\text{exp}}}\) (thick white line).
**Construction of the computational model**

**Procedure**

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Description of the stochastic model

Based on solutions of the Helmholtz equation (2D) in a homogeneous atmosphere

\[ P(x_r, z_r; \omega) = i\pi H_0^{(1)}(kd) + Q_c i\pi H_0^{(1)}(kr) \]
Description of the stochastic model

Based on solutions of the Helmholtz equation (2D) in a homogeneous atmosphere

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- \( P \): complex pressure
- \((x_r, z_r)\): receiver coordinates
- \( \omega, k \): pulsation and wavenumber
- \( d \): source–receiver distance
- \( r \): image source–receiver distance
- \( H_0^{(1)} \): Hankel function of the first kind and order 0
- \( Q_c \): cylindrical reflection coefficient
Description of the stochastic model

Based on solutions of the Helmholtz equation (2D) in a homogeneous atmosphere

\[ P(x_r, z_r; \omega + \Theta) = i\pi H_0^{(1)} \left( \frac{kd}{\Lambda} \right) + \Gamma i\pi H_0^{(1)} \left( \frac{kr}{\Lambda} \right) \]
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Random variables

- \( \Lambda \): time stretching \( \in ]0, +\infty [ \)
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P(x_r, z_r; \omega + \Theta) = i\pi H_0^{(1)} \left( \frac{kd}{\Lambda} \right) + \Gamma i\pi H_0^{(1)} \left( \frac{kr}{\Lambda} \right)
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Random variables

- \( \Lambda, \Gamma \): amount of reflected/transmitted wave \( \in [0, 1] \)
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Based on solutions of the Helmholtz equation (2D) in a homogeneous atmosphere

\[ P(x_r, z_r; \omega + \Theta) = i\pi H_0^{(1)} \left( \frac{kd}{\Lambda} \right) + \Gamma i\pi H_0^{(1)} \left( \frac{kr}{\Lambda} \right) \]

Random variables

- \( \Lambda, \Gamma, \Theta \): frequency shifting \( \in [0, +\infty[ \)
Probabilistic model of $\Gamma$, $\Lambda$ and $\Theta$

### Available information:

- $\Lambda$: time stretching $\in [0, +\infty]$  
- $\Gamma$: amount of reflected/transmitted wave $\in [0, 1]$  
- $\Theta$: frequency shifting $\in [0, +\infty]$
Probabilistic model of $\Gamma$, $\Lambda$ and $\Theta$

Available information:

- $\Lambda$: time stretching $\in ]0, +\infty [$
- $\Gamma$: amount of reflected/transmitted wave $\in [0, 1]$
- $\Theta$: frequency shifting $\in [0, +\infty [$

Information Theory and maximum entropy principle:

- Random variable $\Gamma$:
  
  \[ p_{\Gamma}(\gamma) = 1_{[0,1]}(\gamma) e^{-\mu_0 - \gamma \mu_1 - \gamma^2 \mu_2} \]

- Random variable $\Lambda$ and $\Theta$: **gamma distributions** with parameters $(m_{\Lambda}, m_{\Theta})$ (mean) and $(\delta_{\Lambda}, \delta_{\Theta})$ (dispersion)
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To do

Need to find a relation between scatterers geometry and the stochastic surface parameters with the help of reference solutions.

\[ u(m_H, m_W, m_D, \sigma_H, \sigma_D, \sigma_W) \leftrightarrow w(\mu_0, \mu_1, \mu_2, m_\Lambda, m_\Theta, \delta_\Lambda, \delta_\Theta) \]
Probabilistic model of $\Gamma$, $\Lambda$ and $\Theta$

**Information Theory and maximum entropy principle:**

- Random variable $\Gamma$:
  
  $$p_\Gamma (\gamma) = \mathbb{1}_{[0,1]} (\gamma) e^{-\mu_0 - \gamma \mu_1 - \gamma^2 \mu_2}$$

- Random variable $\Lambda$ and $\Theta$: **gamma distributions** with parameters $(m_\Lambda, m_\Theta)$ (mean) and $(\delta_\Lambda, \delta_\Theta)$ (dispersion)

**To do**

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$$u \left( m_H, m_W, m_D, \sigma_H, \sigma_D, \sigma_W \right) \leftrightarrow w \left( \mu_0, \mu_1, \mu_2, m_\Lambda, m_\Theta, \delta_\Lambda, \delta_\Theta \right)$$
Identification of parameters of distribution functions $\rho_\Gamma (\gamma)$, $\rho_\Lambda (\lambda)$ and $\rho_\Theta (\theta)$

Stochastic inverse problem solved with

- a genetic algorithm...
- ...with a multi-objective function

Minimization of the mean-square norm:

Maximization of the log-likelihood function:
Identification of parameters of distribution functions \( p_\Gamma (\gamma) \), \( p_\Lambda (\lambda) \) and \( p_\Theta (\theta) \)

Stochastic inverse problem solved with

- a genetic algorithm...
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Two objectives

- Minimization of the mean-square norm: minimizes the areas where the experimental observations do not belong to the confidence region of the stochastic model
Identification of parameters of distribution functions $p_\Gamma (\gamma)$, $p_\Lambda (\lambda)$ and $p_\Theta (\theta)$

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Two scatterers morphologies

Case 1:

\((m_H, \sigma_H) = (10, 2) \text{ cm}\)
\((m_W, \sigma_W) = (20, 4) \text{ cm}\)
\((m_D, \sigma_D) = (30, 6) \text{ cm}\)

Case 2:

\((m_H, \sigma_H) = (40, 8) \text{ cm}\)
\((m_W, \sigma_W) = (40, 8) \text{ cm}\)
\((m_D, \sigma_D) = (30, 6) \text{ cm}\)

\(u_1 = (0.1, 0.2, 0.3, 0.2, 0.2, 0.2)\)

\(u_2 = (0.4, 0.4, 0.3, 0.2, 0.2, 0.2)\)
Confidence region from the stochastic model

\[ u_1 = (0.1, 0.2, 0.3, 0.2, 0.2, 0.2) \]

\[ u_2 = (0.4, 0.4, 0.3, 0.2, 0.2, 0.2) \]
Confidence region from the stochastic model

Experimental observations belong to the confidence regions of the stochastic model, independently of the dispersion on the system.
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- based on solutions of the 2D Helmholtz equation,
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...augmented with a probabilistic approach of uncertainties...
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Construction of a stochastic model for propagation over complex surfaces based on...

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...augmented with a probabilistic approach of uncertainties...
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...leading to a statistical, accurate and relevant description of sound fields above the scatterers.
Determination of stochastic reflection coefficients

- Only one source/receiver positions in this example
- Repeating the operation for several incidence angles would allow the determination of stochastic reflection coefficients for inclusion in e.g. ray tracing software
Scatterers geometry

- Scatterers with rectangular cross sections were used. The method is very general and could be used for any scatterers geometry:
  - hemispheres
  - cylinders
  - ...

Perspectives
Thank you for your attention.