Computational model for long-range nonlinear propagation over urban cities

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Shock waves from explosions can damage structures. Signals prediction at long range involves:

- High amplitudes
- Meteorological effects
- Ground effects
- Propagation over urban environments

Development of a computational model for long-range propagation over urban environments.
Shock waves from explosions can damage structures.

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Shock waves from explosions can damage structures.

**Signals prediction at long range involves:**

- High amplitudes
- Meteorological effects
- Ground effects
- **Propagation over urban environments**

Development of a computational model for long-range propagation over urban environments.
Two options

Explicit models (Euler’s equations, ray tracing, …)

- High computational effort
- Model approximations (e.g. diffraction theory)
- Uncertainties on the system parameters (e.g. buildings geometry)

→ May not be the best approach…
Two options

Simplified modeling

- Simplified model based on the Nonlinear Parabolic Equation (NPE) and its extension to propagation over porous ground layers.

  **T. Leissing, P. Jean, J. Defrance, and C. Soize.**
  Nonlinear parabolic equation model for finite-amplitude sound propagation over porous ground layers.

- …associated with a probabilistic approach of uncertainties
Outline

1 Principle and methodology
Outline

1. Principle and methodology

2. Construction of the computational model
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4. Conclusion & perspectives
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Explicit wave propagation model

Random parameters $H, W, D$ modeling a urban city

Probabilistic model depending on a parameter $u$. $u$ depends on the mean values and the dispersions of random variables $H, W$ and $D$.

The urban city parameter $u$ is given.
Explicit wave propagation model

- Source
- Propagation model: City explicitly accounted for

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Simplified wave propagation model

- Source
- Propagation model: over an equivalent city

Random parameters $\Gamma, \Lambda, \Theta$ of the porous medium

Probabilistic model depending on a parameter $w$. $w$ depends on the mean values and the dispersions of random variables $\Gamma, \Lambda$ and $\Theta$.

Identification of the porous medium parameter $w$ is performed by solving a stochastic inverse problem.
Construction of the computational model

Procedure

1. Construction of the probability distribution of urban environments geometrical parameters (height $h$, width $w$, spacing $d$, ...)

Obtain reference solutions

Construct probability distributions of $\Gamma$, $\Lambda$, and $\Theta$, parameters of the porous layer

Identify parameters of $p_{\Gamma}(\gamma)$, $p_{\Lambda}(\lambda)$ and $p_{\Theta}(\theta)$

Solve the stochastic equations with Monte-Carlo
Construction of the computational model

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2. Construction of the computational model
3. Application and validation
4. Conclusion & perspectives
# Construction of the computational model

## Procedure

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Probabilistic model of urban environment geometrical parameters

Information Theory and Maximum Entropy Principle: maximize the uncertainty of the system ("worst-case" scenario)
Probabilistic model of urban environment
geometrical parameters

Information Theory and Maximum Entropy Principle: maximize
the uncertainty of the system (“worst-case” scenario)

Available information concerning $h$, $w$ and $d$:

- Algebraic properties: real, positive quantities
- No dependance between $h$, $w$ and $d$ is assumed
Probabilistic model of urban environment geometrical parameters

Information Theory and Maximum Entropy Principle: maximize the uncertainty of the system ("worst-case" scenario)

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The Maximum Entropy Principle yields

- Independance of random variables $H$, $W$ and $D$
- Probability distributions of $H$, $W$ and $D$ are gamma distributions
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Reference model

Boundary Element Method (BEM)

- 2D simulations
- Perfectly reflecting surfaces (ground, buildings)
- The quantity under interest is

\[ L^{\text{exp}}(\omega) = 10\log_{10}\left(\frac{|P_r^{\text{exp}}(\omega)|^2}{|P_{\text{free}}(\omega)|^2}\right) \]
Reference model

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- 2D simulations
- Perfectly reflecting surfaces (ground, buildings)
- The quantity under interest is

\[
L^{\text{exp}}(\omega) = 10 \log_{10} \left( \frac{\left| P_r^{\text{exp}}(\omega) \right|^2}{\left| P_{\text{free}}(\omega) \right|^2} \right)
\]

**Graph:**

- PLA relative to free field [dB]
- Frequency [Hz]

\[u_2 = (40, 40, 30, 0.2, 0.2, 0.2)\]
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Nonlinear Parabolic Equation model

\[ \Lambda \partial_t R_a + \partial_x \left( \beta \frac{c_0}{2} R_a^2 \right) + \frac{c_0}{2} \int \partial_z^2 R_a \, dx = 0 \text{ (air)} \]

\[ \Lambda \partial_t R_u + \partial_x \left( \beta \frac{c_0}{2} R_u^2 \right) + \frac{c_0}{2} \int \partial_z^2 R_u \, dx = 0 \text{ (urban environment)} \]

\[ \partial_z R_a = \Gamma \partial_z R_u \text{ on the interface} \]

\[ L^{\exp}(\omega) = 10 \log_{10} \left( \frac{\rho_0 c_0^2 \hat{R}_a (\omega + \Theta)}{p_{\text{free}} (\omega + \Theta)} \right)^2 \]
Nonlinear Parabolic Equation model

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\[ L^{\exp}(\omega) = 10 \log_{10} \left( \left| \frac{\rho_0 c_0^2 \hat{R}_a(\omega + \Theta)}{p_{\text{free}}(\omega + \Theta)} \right|^2 \right) \]

Random variables

- \( \Lambda \): time stretching \( \in ]0, +\infty[ \)
- \( \Gamma \): amount of reflected/transmitted wave \( \in [0, 1] \)
- \( \Theta \): frequency shifting \( \in [0, +\infty[ \)
Probabilistic model of $\Gamma$, $\Lambda$ and $\Theta$

Information Theory and Maximum Entropy Principle:

- Random variable $\Gamma$:
  
  $$p_\Gamma(\gamma) = \mathbb{1}_{[0,1]}(\gamma) e^{-\mu_0 - \gamma \mu_1 - \gamma^2 \mu_2}$$

- Random variable $\Lambda$ and $\Theta$: **gamma distributions** with parameters
  
  $m_\Lambda$ (mean) and $\delta_\Lambda$ (standard deviation)
  
  $m_\Theta$ (mean) and $\delta_\Theta$ (standard deviation)
Construction of the computational model

Procedure

1. Construction of the probability distribution of urban environments geometrical parameters (height $h$, width $w$, spacing $d$, . . .)

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Information Theory and Maximum Entropy Principle:

- Random variable $\Gamma$:

$$p_{\Gamma}(\gamma) = 1_{[0,1]}(\gamma) e^{-\mu_0 - \gamma \mu_1 - \gamma^2 \mu_2}$$

- Random variable $\Lambda$ and $\Theta$: **gamma distributions** with parameters $m_\Lambda$ (mean) and $\sigma_\Lambda$ (standard deviation)

  - $m_\Theta$ (mean) and $\sigma_\Theta$ (standard deviation)
Information Theory and Maximum Entropy Principle:

- Random variable $\Gamma$:

\[
p_\Gamma(\gamma) = 1_{[0,1]}(\gamma) e^{-\mu_0 - \gamma \mu_1 - \gamma^2 \mu_2}
\]

- Random variable $\Lambda$ and $\Theta$: gamma distributions with parameters $m_\Lambda$ (mean) and $\sigma_\Lambda$ (standard deviation), $m_\Theta$ (mean) and $\sigma_\Theta$ (standard deviation)

To do:
Find a relation between buildings geometry (height, width, spacing...) and parameters $(\mu_0, \mu_1, \mu_2, m_\Lambda, m_\Theta, \delta_\Lambda, \delta_\Theta)$ with the help of the reference model.
Identification of parameters of distribution functions $p_{\Gamma}(\gamma)$, $p_{\Lambda}(\lambda)$ and $p_{\Theta}(\theta)$

Stochastic inverse problem solved with

- An evolutionary algorithm…
- … with a multi-objective function
Identification of parameters of distribution functions $p_\Gamma(\gamma)$, $p_\Lambda(\lambda)$ and $p_\Theta(\theta)$

Stochastic inverse problem solved with

- An evolutionary algorithm...
- ...with a multi-objective function

Two objectives

- **Minimization of the mean-square norm:** tries to minimize the areas where the experimental observations do not belong to the confidence region of the stochastic model.
Identification of parameters of distribution functions $p_{\Gamma}(\gamma), p_{\Lambda}(\lambda)$ and $p_{\Theta}(\theta)$

Stochastic inverse problem solved with

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Two objectives

- **Minimization of the mean-square norm:** tries to minimize the areas where the experimental observations do not belong to the confidence region of the stochastic model
- **Maximization of the likelihood:** tries to maximize the likelihood between the experimental observations and observations from the stochastic model
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Two urban city morphologies

Low dispersion:

\[(m_H, \sigma_H) = (10, 2) m \]
\[(m_W, \sigma_W) = (20, 4) m \]
\[(m_D, \sigma_D) = (30, 6) m \]

High dispersion:

\[(m_H, \sigma_H) = (40, 8) m \]
\[(m_W, \sigma_W) = (30, 6) m \]
\[(m_D, \sigma_D) = (30, 6) m \]

\[u_1 = (10, 20, 30, 0.2, 0.2, 0.2)\]
\[u_2 = (40, 40, 30, 0.2, 0.2, 0.2)\]
Inverse stochastic problem

Evolutionary algorithm with

- Population of 50 individuals
- Evolution over 50 generations
- Half the population replaced at each generation
Inverse stochastic problem

Evolutionary algorithm with

- Population of 50 individuals
- Evolution over 50 generations
- Half the population replaced at each generation

$u_1 = (10, 20, 30, 0.2, 0.2, 0.2)$

$u_2 = (40, 40, 30, 0.2, 0.2, 0.2)$
Experimental observations belong to the confidence regions of the stochastic model, independently of the dispersion on the system.
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Conclusions & perspectives

Stochastic model for propagation over complex surfaces based on...

...a simple propagation model...

- Based on the Nonlinear Parabolic Equation model
- Fast but far too simplistic to account for the complexity of the real system
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. . . complexified with a probabilistic approach of uncertainties

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**Perspectives**

Large parametric studies, statistical mapping...
Thanks for your attention!