Predicting the acoustic performance of multilayered structures submitted to structural excitation

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Analytical model
CASC Software

- Model and corresponding software developed at CSTB
- Model calculating the acoustic performances of multilayered elements from the material characteristics of layers
- Main acoustic performances calculated:
  - Sound reduction index
  - Impact sound level (ISO taping machine)
  - Absorption coefficient (normal incidence and diffuse field)
  - Rainfall noise (calibrated ISO rain; pr ISO 140-18)
  - Flow induced noise
  - Propagation constant
3 types of layers considered: solid, fluid or porous (Biot’s theory)

Wave approach on infinite multilayered structures using transfer matrix technique

Finite size taken into account using spatial windowing technique
Analytical model
Transfer matrix approach

Multilayered Structure

Layer 1
Layer 2
Layer 3
Layer N

Layer i

Interface 1
Interface 2

\[ U_{1\text{layer } i} = \left[ T_{\text{layer } i} \right] U_{2\text{layer } i} \]
Fluid Layer : 1 wave type

\[
\begin{bmatrix}
    P_1 \\
    V_{z1}
\end{bmatrix}
= \begin{bmatrix}
P_2 \\
V_{z2}
\end{bmatrix}

\begin{bmatrix}
k_z = \cos \theta \frac{\omega}{c_{fluid}} \\
\end{bmatrix}

\begin{bmatrix}
k_z = \sqrt{\left(\frac{\omega}{c_{fluid}}\right)^2 - k_x^2}
\end{bmatrix}

\[
\begin{bmatrix}
P_1 \\
V_{z1}
\end{bmatrix}
= \begin{bmatrix}
\cos(k_z h) & j \omega \rho_{fluid} \sin(k_z h) / k_z \\
j k_z \sin(k_z h) / (\omega \rho_{fluid}) & \cos(k_z h)
\end{bmatrix}
\begin{bmatrix}
P_2 \\
V_{z2}
\end{bmatrix}
\]
Solid Layer : 2 elementary plane wave types

Compressional Wave
\[ \Phi = B_{ip} e^{-jk_{zp} z} + B_{rp} e^{jk_{zp} z} \]

Shear Wave
\[ \Psi = B_{is} e^{-jk_{zs} z} + B_{rs} e^{jk_{zs} z} \]

\[
\begin{bmatrix}
\sigma_z \\
\tau_{xz} \\
V_x \\
V_z
\end{bmatrix} = \begin{bmatrix} B_{ip} \\
B_{rp} \\
B_{is} \\
B_{rs} \end{bmatrix} \begin{bmatrix}
\sigma_{z1} \\
\tau_{xz1} \\
V_{x1} \\
V_{z1}
\end{bmatrix} = \begin{bmatrix} A(z) \end{bmatrix} \begin{bmatrix}
\sigma_{z2} \\
\tau_{xz2} \\
V_{x2} \\
V_{z2}
\end{bmatrix} = [A(z = 0)] [A(z = h)]^{-1} \]
Analytical model

Solid layer transfer matrix

Porous Layer: 2 coupled compressional waves and 1 shear wave propagating simultaneously in both phases (solid phase and fluid phase)

\[
\begin{bmatrix}
\sigma_{z1}^s \\
\tau_{xz1}^s \\
\sigma_{z1}^f \\
V_x^s \\
V_z^s \\
V_f^s \\
V_{z1}^f
\end{bmatrix}
= 
\begin{bmatrix}
T_{porous}
\end{bmatrix}
\begin{bmatrix}
\sigma_{z2}^s \\
\tau_{xz2}^s \\
\sigma_{z2}^f \\
V_x^s \\
V_z^s \\
V_f^s \\
V_{z2}^f
\end{bmatrix}
\]
Analytical model

$\begin{bmatrix}
\mathbf{I}_{\text{layer } i/\text{layer } i+1}
\end{bmatrix}
\mathbf{U}_{2\text{couche } i}
+\begin{bmatrix}
\mathbf{J}_{\text{layer } i/\text{layer } i+1}
\end{bmatrix}
\mathbf{U}_{1\text{couche } i+1} = 0$
- Assembling Transfer and Interface Matrices
- Applying boundary conditions on both sides of multilayered structure
- Solving the system to calculate appropriate index
Analytical model
Structural excitation

Impact noise level on emission side

Emission

Impact (Tapping machine)

Impact noise level on reception side

Reception

Localized mechanical excitation decomposed into propagating normal stress plane waves (using spatial Fourier technique)

Infinite multilayered System

Velocity field at bottom interface (calculated in wave number domain) leads to radiated intensity and impact noise
Taping machine excitation force

Excitation force for frequency bandwidth $\Delta f$ depends on multilayer system input mobility $Y_{input}$:

$$\left| F_e \right|^2 = 4f_s^2 \Delta f \left| \frac{2M_{hammer}v_{0hammer}}{i\omega M_{hammer}Y_{input} + 1} \right|^2$$

$f_s$ : impact frequency of taping machine (10 Hz)
$v_{0hammer}$ : hammer impact velocity (1 m/s)
$M_{0hammer}$ : hammer mass (0.5 kg)

Multilayered system input mobility $Y_{input}$ obtained from inverse Fourier transform

$$Y_{input} = \frac{V_{top\ surface}}{F}$$
Vibrational field decomposed into incident and reflected compressional and shear waves following Snell’s law

\[ k_p \approx \omega / \sqrt{E / \rho} \]

\[ k_s = \omega / \sqrt{G / \rho} \]

- For an isotropic solid: \( E \) estimated from compressional stiffness measurements and \( G \) deduced from

\[ G = E / 2(1 + \nu) \]

- For a non isotropic solid: \( E \) and \( G \) measured separately
Dynamic compressional stiffness of elastic interlayer

- according to standard ISO 9052-3
- stiffness per unit area $s'$ and loss factor estimated near resonance frequency $f_r$ from mobility measurements

\[
f_r = 2\pi \sqrt{\frac{s'}{m'}} \quad \text{and} \quad E = s' h
\]

$m'$: Loading mass per unit area
$s'$: Dynamic compressional stiffness
$h$: Thickness of interlayer
$E$: Elastic modulus
Dynamic shear stiffness of elastic interlayer

- no standard
- stiffness per unit area $r'$ and loss factor estimated near resonance frequency $f_r$ from mobility measurements

$$f_r = \frac{2\pi \sqrt{r' \cdot m'}}{m'}$$
and
$$G = r' \cdot h$$

$m'$: Loading mass per unit area
$r'$: Dynamic shear stiffness
$h$:Thickness of interlayer
$G$: Shear modulus

⚠️ $G$ can depend on direction $x$ or $y$
In standard ISO 9052-3, loading mass of 8 kg for samples of 20x20 cm² (10 cm thick concrete floating floor)

- Loading mass close to actual charge of floating floor preferable
- Thickness of resilient layer under charging mass to be determined
- Dynamic stiffness of resilient layer measured with glue for glued floating system

- Glue has very important effect on resilient layer characteristics: dynamic stiffness increases and damping factor decreases
- Fibrous material characteristics highly modified → easy glue migration into the fibers
Impact noise Prediction/Measurement

- Large number of resilient layers:
  - different materials (cellular or not, porous or not)
  - different thicknesses (from 2-3 mm to 50-100 mm)
- Impact noise reduction $\Delta L$ deduced from impact noise level of the base floor with and without the floating floor system
- Measurements according to ISO 140 part 8
- Predictions from resilient layer measured characteristics
Multilayered System
40 mm concrete floating floor
15 mm dense mineral wool resilient layer (70 kg/m³)
140 mm concrete base floor

Resilient layer
- porous
- flow resistivity measured: 58 kPa s/m²
- Elastic modulus and loss factor (skeleton):
  \[ E = 120 \text{ kPa} \quad \text{and} \quad \eta = 12\% \]

Performance \(\Delta L_w\)
Measured 26 dB
Predicted 26 dB
Multilayered System
60 mm concrete floating floor
70 mm polystyrene resilient layer (24 kg/m³)
200 mm concrete base floor

Resilient layer
- “solid”
- Elastic modulus and loss factor:
  \( E = 10.7 \text{ MPa} \) and \( \eta = 6\% \)

Performance \( \Delta L_w \)
- Measured 16 dB
- Predicted 16 dB
Multilayered System
40 mm concrete floating floor
3.5 mm polyethylene resilient layer (25 kg/m³)
140 mm concrete base floor

Resilient layer
- “solid"
- Elastic modulus and loss factor:
  \[ E = 416 \text{ kPa} \quad \text{and} \quad \eta = 15\% \]
- Shear modulus:
  \[ G = 484 \text{ kPa} \]

Performance \( \Delta L_w \)
- Measured 15 dB
- Predicted with \( E_{\text{measured}} \) 18 dB
- Predicted with \( E_{\text{measured}} \) and \( G_{\text{measured}} \) 16 dB

![Graph showing measurement and prediction of sound reduction](image)
Multilayered System
10 mm oak wood floorboards
2.5 mm fibrous resilient combined with 2 different glue types, modeled as a solid
140 mm concrete base floor

- Fibrous resilient layer quite sensitive to glue as easy migration within the fibers
- Dynamic stiffness of fibrous layer lower when using the MS Polymer glue (ratio of 10) → much better impact noise performance

Performance $\Delta L_w$
- Measured MS Polymer glue 19 dB
- Predicted MS Polymer glue 17 dB
- Measured PU bi-component glue 15 dB
- Predicted PU bi-component glue 13 dB
Multilayered System
From 40 to 100 mm concrete floating floor
70 mm polystyrene resilient layer (24 kg/m³)
200 mm concrete base floor

Predicted Performance $\Delta L_w$
40 mm floating floor $\rightarrow$ 14 dB
50 mm floating floor $\rightarrow$ 15 dB
60 mm floating floor $\rightarrow$ 16 dB
70 mm floating floor $\rightarrow$ 17 dB
80 mm floating floor $\rightarrow$ 17 dB
90 mm floating floor $\rightarrow$ 18 dB
100 mm floating floor $\rightarrow$ 19 dB
Multilayered System
60 mm concrete floating floor
70 mm polystyrene resilient layer (24 kg/m³)
From 120 to 240 mm concrete base floor

Predicted Performance $\Delta L_w$

- 120 mm base floor $\rightarrow$ 18 dB
- 130 mm base floor $\rightarrow$ 19 dB
- 140 mm base floor $\rightarrow$ 19 dB
- 150 mm base floor $\rightarrow$ 18 dB
- 160 mm base floor $\rightarrow$ 17 dB
- 170 mm base floor $\rightarrow$ 17 dB
- 180 mm base floor $\rightarrow$ 16 dB
- 190 mm base floor $\rightarrow$ 16 dB
- 200 mm base floor $\rightarrow$ 16 dB
- 210 mm base floor $\rightarrow$ 15 dB
- 220 mm base floor $\rightarrow$ 15 dB
- 230 mm base floor $\rightarrow$ 15 dB
- 240 mm base floor $\rightarrow$ 15 dB
Prediction / measurement
Rainfall noise

• Laboratory set-up for artificial rainfall noise measurement (“wet” method) has been implemented by CSTB following draft standard ISO/FDIS 140-18 for measurement of sound generated by rainfall on building elements using artificial raindrop

• Analytical tools have to be developed in order to improve design

- 9x7 Grid → 63 holes with ø 1mm
- Re-inforcing Structure
- Filled permanently with 1 cm of water
- Positioned 2.6 m above tested element
Measurement
Heavy rainfall noise

- Holes diameter 1 mm
- Number of holes per unit area 60 m⁻²
- Fall height 3.5 m
- Drop diameter 5 mm
- Impact velocity 7 m/s
- Rainfall rate 40 mm/h

- Inclination 5 deg. for roofs up to 30 deg. for skylights
• Impact force for a drop

\[ F(t) = \rho \pi \frac{D^2}{4} V_t^2 \left\{ 1 - \frac{3V_t t}{4D} \right\} \quad \text{for } 0 \leq t \leq \frac{4D}{3V_t} \]

• Power spectrum of the excitation force associated with a raindrop of diameter \( D \)

\[ P_{\text{exc}}(\omega) = n \left| \tilde{F}(\omega) \right|^2 T_0 = \frac{10^{-3} R}{3600} \frac{6}{\pi D} \left| \tilde{F}(\omega) \right|^2 T_0 \]

- \( T_0 \) : reference time (1s)
- \( R \) : rainfall rate
- \( n \) : number of drops impacting per unit area of roof per unit time for rainfall rate \( R \)

• For flexible structures, impact force spectrum depends on structure input mobility \( Y_{\text{input}} \)

\[ \tilde{F}_{\text{flex}}(\omega) = \frac{Y_{\text{drop}}}{Y_{\text{input}} + Y_{\text{drop}}} \tilde{F}(\omega) \quad \text{with } Y_{\text{drop}}, \text{flow mobility of the water drop} \]

\[ Y_{\text{drop}} = \frac{4}{\rho V_t \pi D^2} \]
Rainfall noise Roofing system

- System surface: 1.18x1.08 m²
- Mounting slope: 5 deg.
- Rainfall excitation centered

Performance $L_{IA}$
- Measured 70 dB(A)
- Predicted 70 dB(A)
Prediction / measurement

Rainfall noise

Roofing system

Aluminum (0.72 mm)

Polystyrene (50 mm)

Heavy Layer

(2 mm – 4 kg/m²)

Performance $L_{IA}$

Measured 70 dB(A)

Predicted 70 dB(A)

Predicted with heavy layer 60 dB(A)
Conclusions

• Prediction model has been developed to investigate the behavior of infinite multilayered structures for structural excitation

• Prediction tool is developed to complement experimental measurements following international standards implemented at CSTB laboratory LABE

• Prediction model can be used to help industrial partners to improve their products or to develop new products

• Industrial partners have gained a better knowledge of their products and systems