A homogenised vibratory model for predicting the acoustic properties of hollow brick walls

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A B S T R A C T

The prediction and the physical understanding of sound transmission through masonry walls made of hollow bricks remain an open question. To solve this problem a semi-analytical approach is proposed. The inhomogeneous structures of the brick wall are homogenised and a simplified analytical model is established to calculate the transmission loss of an equivalent finite and multilayered anisotropic plate. An efficient numerical homogenisation technique is derived to define the equivalent anisotropic brick. This process only needs the knowledge of the elastic tensor of the brick material that has been determined using ultrasonic measurements. The features of the simplified brick wall have then investigated through Lamb waves dispersion curves. Finally, the model has been used to explain the transmission loss curve of a wall and a good agreement between predictions and test data is obtained.

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1. Introduction

Hollow bricks are widely used as building structural elements. Their thermal and acoustic properties are greatly influenced by the alveolar pattern of the blocks. The main purpose of these alveolar structures is to increase the thermal insulation properties of the walls as well to decrease its weight. Typical geometry is shown in Fig. 1 with periodic or quasi-periodic patterns that are of common use. They obviously lead to particular acoustic behaviours. The aim of the present work is to explore the physical phenomena that are involved in the acoustic properties of such building elements.

Accurate prediction of the acoustic properties of a hollow brick wall yields several difficulties. The problem is multiscaled since three relevant spatial scales can be distinguished: the first one is the brick material used that is to say the fired clay medium. The second one is the block itself while the last one is the whole masonry wall built from the periodical assembling of blocks with joints and plaster layer as shown in Fig. 2.

Scholl and Weber [1], through experimental observations, pointed out the importance of parameters such as thickness and core geometry on the acoustic properties of the wall. Schneider and Fischer [2] measured the sound transmission through a hollow brick wall where the holes were filled with absorbing materials. This study pointed out the crucial...
purpose of the losses to improve the sound insulation of such walls. The modal behaviour of separated blocks with various hole patterns has been also investigated by Weber and Buckle [3].

From a theoretical point of view, only a few studies have been performed on the acoustic properties of brick walls. Maysenholder’s studies [4–6] on the acoustic transmission through periodically inhomogeneous plate must be mentioned. Analytically taking into account the double periodicity of masonry block walls (alveolar scale in the brick and brick in the wall), one of his previous work [6] clearly showed that the precise calculation of the acoustic transmission is too time consuming.

In spite of the recent progress made concerning the numerical modelling of sound transmission through walls [7–9], in the case of hollow blocks, these numerical techniques are still limited to the low frequency range, especially for thick hollow brick walls [10].

Therefore, in this paper, a simpler approach is proposed: an infinite thick orthotropic plate model is used [11]. The finite size of the wall is then taken into account using the spatial filtering technique developed by Villot et al. [12], and Villot and Guigou [13]. In order to get the elastic properties of the partition, a homogenisation technique applied to one brick block is derived.

The present study is divided into four parts. In Section 2, the thick plate model is described. In Section 3, the hollow block homogenisation process is presented: a numerical model of an alveolar brick is used to determine the elastic tensor of the equivalent anisotropic material. This numerical model requires the knowledge of the elastic properties of the brick (fired clay for instance) material. These parameters are experimentally determined (Section 4) from material samples. In Section 5, the application of this model to a 20 cm hollow brick wall is presented and results are compared to measured data. This particular example is used to highlight the physical phenomena responsible for the acoustic transmission through hollow brick masonry.

2. Thick plate model of a hollow brick wall

The alveolar brick wall is assimilated to an infinite thick homogeneous and anisotropic plate. An analytical model can be developed to calculate the sound transmission loss.

To achieve it, we consider a plate of thickness $h$, and dimensions $(L_x, L_y)$ in the $(x, y)$ plane embedded in air (density $\rho_a$ and speed of sound $c_a$).

Using a spatial Fourier transform of the elasticity equation [14], Skelton and James [11] established a matrix relation between stresses and displacements vectors of the upper and lower surfaces of the plate:

$$T_{0 \times 1} = S_{6 \times 6}(k_x, k_y) U_{0 \times 1}$$

with

$$T = (\hat{T}_{zz}(k_x, k_y, z = 0), \hat{T}_{zz}(k_x, k_y, z = 0), \hat{T}_{zz}(k_x, k_y, z = 0), \hat{T}_{pp}(k_x, k_y, z = 0), \hat{T}_{pp}(k_x, k_y, z = h), \hat{T}_{pp}(k_x, k_y, z = h))^T$$

$$U = (\hat{u}_z(k_x, k_y, z = 0), \hat{u}_y(k_x, k_y, z = 0), \hat{u}_z(k_x, k_y, z = 0), \hat{u}_y(k_x, k_y, z = h), \hat{u}_y(k_x, k_y, z = h), \hat{u}_z(k_x, k_y, z = h))^T$$
where \( k_x, k_y \) are the wavenumbers along the \( x \) and \( y \) directions (respectively), \( \hat{T}_{zi}, \hat{u}_i, i = x, y, z \) are the Fourier transform of the stress element \( T_{zi} \) and the displacement component \( u_i \), while \( S \) is a \( 6 \times 6 \) matrix also called “spectral stiffness matrix”.

In the case of a multilayered wall, each layer \( j \) is characterised by its matrix \( S_j \). The transfer matrix formalism allows us to connect normal stresses and displacements vectors of each layer through the transfer matrix \( M \) of the system \([15,16] \):

\[
[\psi]_{6 \times 1}^0 = M_{6 \times 6} [\psi]_{6 \times 1}^b = \prod_{j=1}^N M_j [\psi]_{6 \times 1}^b.
\]

where

\[
\psi = (\hat{T}_{zi}(k_x, k_y, z), \hat{T}_{zy}(k_x, k_y, z), \hat{T}_{zx}(k_x, k_y, z), \hat{u}_x(k_x, k_y, z), \hat{u}_y(k_x, k_y, z), \hat{u}_z(k_x, k_y, z))^T
\]

This formalism can be used to study the transmission through a brick wall (first layer) with an additional plaster lining (second layer).

Neglecting air viscosity, the boundary conditions on tangential stresses \( \hat{T}_{ji} = 0 \) at \( z = 0, h \) reduce Eq. (1) to a simple \( 2 \times 2 \) matrix system:

\[
\begin{pmatrix}
\hat{T}_{zz}(z = 0) \\
\hat{T}_{zz}(z = h)
\end{pmatrix} = L_{2 \times 2} \begin{pmatrix}
\hat{u}_z(z = 0) \\
\hat{u}_z(z = h)
\end{pmatrix}
\]

with \( L \) a \( 2 \times 2 \) matrix whose elements are \( S_{ij}, (i, j = 1, ..., 6) \) dependent.

If the plate is excited by a diffuse field at \( z = 0 \) (see Fig. 3), the sound transmission coefficient is evaluated as in \([17]\)

\[
\tau_{\text{diffuse}} = \frac{1}{\pi} \int_{0}^{\theta_f} \int_{0}^{\phi_f} \tau(\theta, \phi, f) \sin(\theta) \cos(\theta) \, d\theta \, d\phi
\]

\[
\tau(f, \theta, \phi) = |T|^2, \quad T = \frac{2ZL_{21}}{L_{12}L_{21} - (L_{11} + Z)(L_{22} - Z)}, \quad Z = \frac{\rho_c c_a}{\cos(\theta)}
\]

\[0 \leq \varphi \leq 2\pi, 0 \leq \theta \leq \frac{\pi}{2}\] (see Fig. 3)

A drawback of this approach comes from the finite dimensions of the wall in the \((x, y)\) plane which are not taken into account. For building-acoustics field (where typical dimensions are about few metres), this mainly impacts the low frequency range (typically 100–500 Hz) \([18]\). To avoid this drawback, it is possible to use the spatial filtering technique that introduces the diffraction effects related to the finite size of the structure.

Then the transmission coefficient becomes

\[
\tau_{\text{finite}}(f, \theta, \phi, L_x, L_y) = \tau(f, \theta, \phi) \sigma(k_a \sin(\theta), \phi, L_x, L_y) \cos(\theta)
\]

where

\[
\left\{\begin{array}{l}
k_a = \frac{2nf}{c_a} \\
\sigma = \frac{L_x L_y}{\pi^2} \int_{0}^{k_a} \int_{0}^{2\pi} \frac{1 - \cos((k_l \cos(\zeta - k_a \sin(\theta \cos \phi) L_x)) 1 - \cos((k_l \sin(\zeta - k_a \sin(\theta \sin \phi) L_y))}{[k_l \cos(\zeta - k_a \sin(\theta \cos \phi) L_x)]^2} dx \, dk_l \int_{0}^{2\pi} \frac{1 - \cos((k_l \sin(\zeta - k_a \sin(\theta \sin \phi) L_y))}{[k_l \sin(\zeta - k_a \sin(\theta \sin \phi) L_y)]^2} dx \, dk_l
\end{array}\right.
\]

The \( \sigma \) function corresponds to the radiation efficiency of the spatially windowed plate \([17]\). The explicit expression of the transmission loss (TL) can be deduced:

\[
\text{TL}(f) = -10 \log(\tau_{\text{finite}})
\]

Fig. 3. Sound transmission through a homogenised plate: a superposition of sound plane waves are incident on the structure.
It should be noted that the $L_{ij}$ coefficients mainly depend on the total loss factor of the wall (i.e. the sum of internal, radiation and coupling loss factors) and its elastic tensor. Both appear in the coefficients $L_{ij}$ since they are related to $C_{2\beta}(1-\eta)$, $i,j = [1,2]$: $x, \beta = [1,6].$

The total loss factor can be quite easily obtained through measurements [19,20]. However, to our knowledge, it is much more difficult to obtain the $C_{2\beta}$ tensor for inhomogeneous building elements such as hollow bricks. This is why the homogenisation process of the following section is proposed.

3. Homogenisation process

The starting point to study the acoustic properties of hollow brick is the theory of elasticity in inhomogeneous media. The displacement field is governed by the following wave equation [21]:

$$\rho(r) \frac{\partial^2 u(r,t)}{\partial t^2} = \frac{\partial^2 u(r,t)}{\partial x_i^2} + \frac{\partial^2 u(r,t)}{\partial x_j^2}$$

where the summation convention has been adopted and $C_{ijkl}(r)$ are (respectively) the components of the elastic tensor, mass density and displacement at each point $r$ of the medium, $i,j,k,l = [1,3]$. Assuming an infinite and periodic medium along the three directions, the solution of Eq. (7) is obtained through a Fourier series of the physical variables (according to the Bloch theorem [22]).

In the limit of long wavelengths, Maysenholder [4] succeeded in analytically defining the elastic tensor $<C_{2\beta}>$ of an equivalent anisotropic medium but the whole process is relatively heavy. That is the reason why an alternative numerical expansion is used, based for instance on the finite element method (FEM) applied to one block. Thus the computation time is negligible, the whole process is relatively heavy. That is the reason why an alternative numerical expansion is used.

In the method used, various mechanical loadings applied to a hollow brick are numerically simulated. Thereafter, we assume that a moderate anisotropy (orthotropy) is enough to describe the alveolar block. Thus, the equivalent material is defined by nine mechanical parameters: 3 young modulus, 3 shear modulus and 3 Poisson coefficients.

3.1. Young modulus and Poisson ratios computation

Let us consider a test along the $X$-axis with the following boundary conditions applied to the brick $(U = (U_x, U_y, U_z)^T$ is the displacement field):

$$\left\{ \begin{array}{l}
U_x(x=0,y,z) = U_y(x,y=0,z) = U_z(x,y,z=0) = 0 \\
U_x(x = l_x,y,z) = -1
\end{array} \right.$$  (8)

In this case, the Young modulus along $X$ is written (see Fig. 4):

$$E_X = - \frac{R_{Xx}}{l_x l_y}$$  (9)

where $R_{Xx}$ is the total force reaction on the face $x = l_x$ (numerically obtained) and $l_x, l_y, l_z$ are the hollow brick dimensions along $X$, $Y$ and $Z$ (respectively).

Poisson coefficients associated to the $XY$ and $XZ$ planes are

$$v_{XY} = - \frac{\partial u_y}{\partial x} = - \frac{U_y l_x}{l_y} \quad \text{and} \quad v_{XZ} = - \frac{\partial u_z}{\partial x} = - \frac{U_z l_x}{l_z}$$  (10)

where $\partial u_x, \partial u_y$ are, respectively, the apparent deformations along the $X$ and $Y$ axes.

![Fig. 4. Displacement field imposed to obtain $E_X$. The induced displacements $U_x(y = l_y)$ and $U_z(z = l_z)$ are used to compute the Poisson coefficients $v_{XY}, v_{XZ}$.](image-url)
The same procedure applied along the two other directions provides the Young modulus $E_Y$, $E_Z$ and the last independent Poisson coefficient $\nu_{YZ}$ (note that the symmetry of the elastic tensor implies the relation $\nu_{ij}E_j = E_i\nu_{ji}$, $i,j = X,Y,Z$). Thus six of the nine elastic constants are obtained thanks to compression tests.

3.2. Shear modulus computation

Shear loadings are more difficult to perform because only shear stresses $T_{ij}$; $i \neq j$; $i,j = 1,2,3$ must be generated during the test.

Considering a shear experiment in the $XZ$ plane, the displacement field imposed is (see Fig. 5)

$$
\begin{aligned}
U_Y(x,y,z = 0) &= 0, \quad U_Z(x,y,z = 0) = x \\
U_Y(x,y,z = l_z) &= l_z, \quad U_Z(x,y,z = l_z) = U_Z(x,y,z = 0) = 0 \\
U_Z(x = 0,y,z) &= 0, \quad U_Z(x = 0,y,z) = z \\
U_Z(x = l_x,y,z) &= l_x, \quad U_X(x = l_x,y,z) = U_X(x = 0,y,z) \\
U_Y(x,y = 0,z) &= 0
\end{aligned}
$$

(11)

By definition, the shear modulus $G_{XZ}$ is

$$
G_{XZ} = \frac{T_{XZ}}{\gamma_{XZ}}
$$

(12)

where $T_{XZ}$ and $\gamma_{XZ}$ are, respectively, the shear stress in the $XZ$ plane and the distortion angle induced by the shearing. With these imposed boundary conditions, one has

$$
\gamma_{XZ} = 2G_{XZ} = \frac{\partial U_Y}{\partial z} + \frac{\partial U_Z}{\partial x} = \alpha + \beta
$$

$$
\alpha = \tan(z) = \frac{U_Z(x = l_z)}{l_z} = \frac{U_Z(x = l_x)}{l_x} = \tan(\beta) \approx \beta = 1
$$

(13)

Finally,

$$
G_{XZ} = \frac{T_{XZ}}{2}
$$

(14)

Applying the same reasoning in the two other planes, the shear modulus $G_{XY}$ and $G_{YZ}$ of the homogenised brick are calculated. Therefore the elastic tensor of the orthotropic homogenised block can be explicitly evaluated as

$$
C = \begin{pmatrix}
C_{11} &= \frac{E_x(1+\nu_{xy})}{4} & C_{12} &= \frac{E_x(1+\nu_{xy})}{4} & C_{13} &= \frac{E_x(1+\nu_{xy})}{4} & 0 & 0 & 0 \\
C_{12} &= \frac{E_x(1+\nu_{xy})}{4} & C_{22} &= \frac{E_x(1+\nu_{xy})}{4} & C_{23} &= \frac{E_x(1+\nu_{xy})}{4} & 0 & 0 & 0 \\
C_{13} &= \frac{E_x(1+\nu_{xy})}{4} & C_{23} &= \frac{E_x(1+\nu_{xy})}{4} & C_{33} &= \frac{E_x(1+\nu_{xy})}{4} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} &= G_{yz} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} &= G_{xz} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66} &= G_{xy} &
\end{pmatrix}
$$

(15)

with $A = -1 + \nu_{yz} + \nu_{xy} + 2\nu_{xz} + \nu_{xy} + 2\nu_{yz} + \nu_{xz} + \nu_{xy} + \nu_{yz} + \nu_{xz}$.

It may be noted that two distinct causes are responsible for the anisotropy of the hollow brick. The first one is a “geometric” anisotropy directly linked to the complex alveolar pattern of the structure. The second one is connected to the...
elastic behaviour of the brick material itself. Clearly, the precise knowledge of the material properties (density and elastic constants) is a key step to correctly simulate the hollow brick and thus the acoustic properties of such masonry walls.

4. Parameters of the model

Very few studies deal with the mechanical properties of brick material. One example is given by the work of Huon et al. [23] where static measurements on samples show a strong anisotropic behaviour. Actually, due to its lamellar microscopic structure and the extrusion forming process of hollow blocks [24], the clay material can be considered as a transverse isotropic solid with axis of symmetry along the z direction. The ultrasonic method, based on the measurement of flight times in the sample [25], allows us to specify the different elastic constants. The values measured on a sample that is coming from one specific brick (see Fig. 6) are given in Table 1:

\[
\delta C_{ij} = \sum_t \left[ \frac{\partial^2 C_{ij}}{\partial t^2} \right] \delta x = \left( \frac{\delta t}{t} \right)^2 \delta \rho + \left( 2 \frac{\delta d}{t^2} \right) \delta d + \left( 2 \frac{\delta d}{t} \right) \delta t
\]

with \(d, t, \rho, \delta d, \delta t, \delta \rho\) are the propagation distance and the flight time of the acoustic mode considered, the mass density and the uncertainties associated (respectively).

The brick material is characterised by a soft direction \((C_{33} < C_{22})\) which is perpendicular to the clay layers (Fig. 6a). This also explains why the shear modulus out of the isotropic plane is weak compared to the other ones \((C_{44} = C_{222} < C_{66} = C_{333})\).

It should be noticed that during the homogenisation process the confined air in the holes is neglected and substituted by vacuum. This point is widely justified because of the high compressibility of air compared to brick material [26].

For the brick studied here (Fig. 6b), the elastic tensor of the orthotropic equivalent hollow block has the following components:

\[
C = \begin{pmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\
C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & C_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & C_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & C_{66}
\end{pmatrix}_{\text{GPa}}
\]

These values are consistent with the alveolar pattern considered (see Fig. 6b). First of all, the relation \(C_{11} \approx C_{33}\) is the result of the quasi isotropic behaviour in the \((X, Z)\) plane. The elastic constant \(C_{22}\) is the highest one because the \(Y\) direction coincides with the extrusion direction of the clay material. Furthermore, in that direction, the stiffness of the brick and the clay are related by \(E^\text{brick} = S_{\text{clay}}(h_{l_k})^{-1} E^\text{clay}\) (if \(E_{Y}\) is Young modulus and \(S_{\text{clay}}\) the surface taken up by the clay material, \(S_{\text{clay}} \ll h_{l_k}\)). In the same way, the high softness of the hollow core in the \((X, Z)\) plane naturally leads to a low shear modulus \(G_{XZ} \equiv C_{55}\).

![Fig. 6. (a) Local frame associated with the clay material (sample's volume equals a few cm³). The \((X, Y)\) plane matches with the isotropic plane while the \(Y\) direction is the extrusion one and (b) picture of the hollow brick under consideration \((l_k = 56\text{cm}, l_y = 27.5\text{cm}, h = 20\text{cm} \text{ and } m = 18\text{kg})\).](image)

### Table 1

Data obtained on a fired clay sample \((C_{12} = C_{1} - 2C_{66} = 3.2\text{GPa})\).

<table>
<thead>
<tr>
<th>Density (kg/m³)</th>
<th>(C_{11} - 2C_{66}) (GPa)</th>
<th>(C_{12}) (GPa)</th>
<th>(C_{13}) (GPa)</th>
<th>(C_{44}) (GPa)</th>
<th>(C_{55}) (GPa)</th>
<th>(C_{66}) (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1860 ± 5</td>
<td>11 ± 0.5*</td>
<td>3.7 ± 0.3*</td>
<td>1.5 ± 0.7*</td>
<td>2 ± 0.2*</td>
<td>3.9 ± 0.4*</td>
<td></td>
</tr>
</tbody>
</table>

* Where the uncertainties have been calculated according to Eq. (16).
The very low stiffness of the brick compared to classical other building materials (concrete blocks, timber, etc.) must be emphasised because it naturally raises the following issue: how do these peculiarities affect the acoustic radiation of a hollow brick wall?

5. Experimental validation

5.1. Building acoustics and Lamb modes

The problem of sound transmission through plates is of major interest in building acoustics since many partitions can be roughly approximated by this kind of structure. Then a very common low frequency approximation is to only consider bending waves [17]. However, from a theoretical point of view, the plate is an elastic waveguide and, except for quasi normal incidence, an oblique incident acoustic wave generates compression and shear waves in the plate [14]. In the isotropic case, Lamb [27] showed that the reflection of these waves on the upper and lower surfaces of the plate lead to an infinite discrete set of dispersive plate modes called Lamb modes. Nayfeh and Chimenti [28] studied the case of Lamb waves in a general anisotropic plate.

In the present case, the Lamb modes are also called leaky Lamb modes because the wall is immersed in a fluid [29]. However, the plate modes are almost unaffected by the fluid because air density is negligible compared to the brick material one. Thus the following boundary conditions must be verified (air viscosity is neglected):

\[ T_{xz}(x,y,z = \pm \frac{h}{2}) = C_{ijkl} \frac{\partial u}{\partial x_k} \cong 0 \]  \hspace{1cm} (18)

where \( T_{xz} \) is the stress component, \( u \) the displacement field and \( i,k,l = x,y,z \).

For an orthotropic plate (of density \( \rho \), thickness \( h \) and elastic tensor \( C_{ijkl} \)), this condition leads to the dispersion curves, which are obtained solving the following system [28]:

\[
\begin{align*}
F(\rho, C_{ijkl}, v_{Sn}) \sin \left( \frac{n \pi}{h_0} \Phi(\rho, C_{ijkl}, v_{Sn}) \right) \cos \left( \frac{n \pi}{h_0} \Xi(\rho, C_{ijkl}, v_{Sn}) \right) - G(\rho, C_{ijkl}, v_{Sn}) \sin \left( \frac{n \pi}{h_0} \Phi(\rho, C_{ijkl}, v_{Sn}) \right) \cos \left( \frac{n \pi}{h_0} \Xi(\rho, C_{ijkl}, v_{Sn}) \right) & = 0 \\
F(\rho, C_{ijkl}, v_{An}) \sin \left( \frac{n \pi}{h_0} \Phi(\rho, C_{ijkl}, v_{An}) \right) \cos \left( \frac{n \pi}{h_0} \Xi(\rho, C_{ijkl}, v_{An}) \right) - G(\rho, C_{ijkl}, v_{An}) \sin \left( \frac{n \pi}{h_0} \Phi(\rho, C_{ijkl}, v_{An}) \right) \cos \left( \frac{n \pi}{h_0} \Xi(\rho, C_{ijkl}, v_{An}) \right) & = 0 \\
\cos(\Omega(\rho, C_{ijkl}, v_{ShL}, f) h) \sin(\Omega(\rho, C_{ijkl}, v_{ShL}, f) h) & = 0 \Rightarrow v_{ShL}(f) = \frac{c_{SN}}{\sqrt{\frac{\rho}{C_{ijkl} \cdot \rho^2}}}, \quad n = 0, 1, 2, \ldots
\end{align*}
\]  \hspace{1cm} (19)

where \( v_{An}, v_{Sn}, v_{ShL} \) are the phase velocities associated with the three plate modes family and \( F, G, \Phi, \Xi \) and \( \Omega \) functions depend on the plate parameters and those velocities (see Ref. [28] for more details).

The two first equations of that system define the symmetric \( (S_n, n = 0, 1, 2, \ldots) \) and antisymmetric \( (A_n, n = 0, 1, 2, \ldots) \) Lamb modes whereas the last one expresses the phase velocity of the shear horizontal (SH) modes. Application to the orthotropic homogenised brick wall studied above (see Fig. 6(b) and Eq. (17)) is shown in Fig. 7. This result is related to the approximate transverse isotropy of the homogenised material (see Eq. (17)). Actually, the cut-on frequency of the \( S_1 \) mode is independent of the propagation direction (see Section 5).

The question of the connection between these dispersion curves and the acoustic transmission loss of the wall is discussed in next part.

5.2. Practical application

The acoustic properties of a masonry wall made of hollow blocks described in Section 4 are investigated. Small horizontal bed joints are used to build the wall (see Fig. 8a). In order to measure its transmission loss (TL), the wall is placed between two rooms of dimensions 4.22 × 3.35 × 5.56 and 4.22 × 2.5 × 5.92 m³. The total mass per unit area measured for the wall recovered by one plaster layer is about 125 kg/m². The total loss factor \( \eta \) is deduced from structural reverberation times measured on the wall. Its frequency dependence is given in Fig. 8b. A global decrease of the total loss factor with frequency is clearly observed. This is consistent with the fact that both edge and radiation loss factors are mostly important in low frequencies [19]. In the range 50–500 Hz, it also exhibits strong fluctuations probably due to the modal behaviour of the wall. For higher frequencies, the loss factor becomes close to the internal loss factor.

The formalism of section 2 is applied; the transfer matrix of the wall is \( M_{\text{total}} = M_{\text{brick}} \times M_{\text{plaster}} \) with \( M_{\text{brick}} \) the transfer matrix of an orthotropic layer whose thickness is 20 cm and mass density 590 kg/m³. As the plaster is isotropic, \( M_{\text{plaster}} \) stands for the transfer matrix of a 1 cm thick layer whose properties are \( E = 5 \) GPa, \( v = 0.2 \) and \( \rho = 1500 \) kg/m³.
It should be observed that the horizontal joints between blocks have not been taken into account during the homogenisation modelling. In other words, the elastic tensor derived above (see Eq. (17)) is the one used to represent the homogenised structure. Besides, vertical joints are empty in this experiment: the interlocking of blocks is assumed to produce a good continuity from one brick to the other. This hypothesis is consistent with the homogenised plate model used (where such discontinuities are not taken into account).

**Fig. 7.** Dispersion curves of a 20 cm thick homogenised brick wall: (a, b) lamb modes and SH modes propagating along the X direction. (c, d) lamb modes and SH modes along the Y direction.

**Fig. 8.** (a) Masonry wall of dimensions $4.2 \times 2.4 \times 0.21 \text{ m}^3$ along $X$, $Y$, $Z$ (respectively) and (b) total loss factor measured on the wall under study.
Fig. 9 shows the TL measured in laboratory and the calculated one. Both of them are expressed in one-third-octave bands in the frequency range 100–5000 Hz.

The over-all agreement is satisfactory except in very low frequencies (around 100 Hz) where important differences are observed. Two possible reasons may explain it. Because of the modal behaviour of the emission and reception rooms, the diffuse field approximation may not be justified [30], and the modal structure of the whole masonry is not considered in our model.

Moreover, higher in frequency (around 200–300 Hz), the drop observed in the TL is well predicted. Actually, this dip is a usual feature of a bending wave in an anisotropic homogeneous plate. It is well known that the transmission coefficient strongly increases when the speed of bending waves equals the speed of acoustic wave in the surrounding medium (air). In the case of an orthotropic plate, this “coincidence effect” defines a critical region [31] bounded by the two following frequencies:

\[
  f_{cx} = \frac{c^2}{2\pi} \sqrt{\frac{\rho_s}{B_x}} \quad \text{and} \quad f_{cy} = \frac{c^2}{2\pi} \sqrt{\frac{\rho_s}{B_y}}
\]

where \(B_x = \frac{h}{12} (C_{11} - (C_{13}^2/C_{33}))\), \(B_y = \frac{h}{12} (C_{22} - (C_{23}^2/C_{33}))\) are the bending stiffness of the homogenised plate along the x and y axes, and \(\rho_s\) the mass per unit area of the wall.

For the wall studied, numerical calculation roughly localises the critical region between 160 and 350 Hz.

Another decrease of the sound transmission loss appears between 2 and 2.5 kHz. Such dip cannot be explained with a simple thin plate model since the basic assumption of such theory is to limit the analysis to the radiation of the fundamental antisymmetric lamb mode A0. Obviously, this low frequency approximation depends on the plate’s parameters (thickness, mass density and elastic constants). This point explains why a general model for arbitrary thickness homogenised wall has been previously considered (Section 2). Therefore leaky Lamb waves exist in the plate and one mode may radiate in air depending on its Spatial shape [14,29]. Because of the normal displacement shape associated with the symmetric S1 Lamb mode, this mode may contribute to sound transmission from its cut-on frequency, given by the following equation [14]:

\[
  f_{S1}^{\text{cut}} = \frac{C_{33}}{2h} \sqrt{\frac{1}{\rho_s}} = \frac{1}{2} \sqrt{\frac{C_{33}}{\rho h}}
\]

Knowing that \(\rho_s \approx 125 \text{ kg/m}^2\), \(h \approx 21 \text{ cm}\) and \(C_{33} \approx 0.45 \text{ GPa}\) for the hollow brick wall considered, one obtains \(f_{S1}^{\text{cut}} \approx 2200 \text{ Hz}\) (see also Fig. 7(a) and (c)), namely in the range where the transmission loss drops (see Fig. 9). One should also mention that above this cut-on frequency, the mechanism of sound transmission is complex since it results from a superposition of Lamb modes [32].

Finally, the hollow brick wall cannot be seen as a thin homogeneous plate on the entire spectrum (100–5 kHz). Because of the very low stiffness of the block in the thickness direction, such walls display thickness resonances corresponding to the radiation of higher order Lamb modes (i.e. the S1 mode).

6. Conclusion

The problem of sound transmission through multialveolar fired clay walls is addressed. Because of the complex structures involved, a simplified modelling has been chosen. Thus a hybrid method coupling analytical modelling for the
transmission loss of a finite and multilayered anisotropic thick plate with the FEM to homogenise the hollow block has been proposed. The accuracy of the model strongly depends on the experimental characterisation of two key parameters: the elastic tensor of the basic constituent (i.e. the clay material) and the structural loss factor of the masonry wall. The analysis of the transmission loss curves tends to show that hollow blocks walls exhibit usual attributes of homogeneous plate in the spectrum (100–5 kHz). In the low frequency range, the anisotropic behaviour of the wall leads to an increase of its sound transmission coefficient in the critical region. Another interesting point is to remark that such walls are subjected to thickness resonance in the audible range of interest. In practice, this phenomenon is mainly induced by the hollow structure and its high softness in the thickness direction. This study calls for additional investigations, in particular, concerning the effect of joints on sound transmission or the designing of more efficient soundproofing brick walls.

References
