A 2.5D BEM Model for Ground-Structure Interaction

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ABSTRACT

The study of ground-structure interaction due to surface waves (trains or tramways) has been undertaken using a full Boundary Element Method (BEM) approach, both for the ground and structure. Rather than using a 2D Green's function, a 2.5D Green's function is written for a given wave number along the 'infinite' direction so that this method can be considered as a mixture of BEM and wave analysis. Fourier-like integrations of the solutions, for different wavenumbers, yield solutions for point or incoherent line excitations. In the case of railway-like sources, the 2.5D model allows coincidence effects to be considered in, so that the effect of the source position along the track relative to the structure and of the angle of incidence of in-coming waves can be estimated. Also the 2.5D approach gives much faster calculations than a full 3D implementation. The approach is used in a study of ground and foundation interaction, without and with isolating linings.

1. INTRODUCTION

Large cities have to face traffic problems as well as associated issues such as pollution. Several means of reducing or limiting the use of private cars have been attempted with limited success. An alternative is the development of rail transportation. Tramways, for instance [1], which at one time were very popular, were largely suppressed in France, in the course of the last century, in favour of car transportation. The advantages of tramways have been 'rediscovered' in the 1980's and many French towns such as Grenoble, Nantes, and Strasbourg have re-installed them. Such surface rail transportation generate ground vibrations which once transmitted to near-by buildings will propagate and generate undesirable noise.

Such problems can be dealt with in several ways. In [2-4] the use of isolating layers between ground and foundations has been analysed. A 2D FEM/BEM approach has been used and results compared with measurements. There are discrepancies between
computed and measured values above the mass-spring frequency of the interlayer. The measured attenuations were insignificant whereas computed values followed a classical rise after this frequency. Several hypotheses have been offered to account for these differences. Measurements of transfer functions between surface and foundation vibrations, due to tramways also has led to significant differences; measured velocity differences were greater than calculated values.

In considering these discrepancies, the validity of the model employed has been questioned. First, the ground has been in all cases modelled as a homogeneous isotropic half space with characteristics obtained from measurements [14]. Second, the contact between ground and foundation has been assumed to be perfect. Third, a 2D FEM/BEM model was initially employed. This last point has been judged to be the most influential and questionable. A 2D model assumes a 2D geometry and a cylindrical type of excitation where incident waves can only impinge normally on the structures. Consequently, this precludes all coincidence effects. It also implies that sources are coherent line sources, whereas actual trains should rather be treated as incoherent line sources.

At present, the use of 3D FEM/BEM models are restricted to simple problems, such as slabs-on-grade supporting machines, since more complex problems are beyond the scope of PC computers. In the case of lengthy buried walls, an important aspect is likely to be the source part of the 3D problem. In this respect, the use of a 2.5D approach seems to be a practical compromise. Of course, for short walls or piled foundations the full dimensions of the embedded structure must be considered. In a 2.5D analysis, it can be assumed that the geometry considered remains constant along a given direction, whereas the excitation can become point-like or composed of a sum of point excitations, such as an incoherent line. Such an approach was proposed by Duhamel [7] and later used by Jean et al. [8] for the study of noise barriers. Recently, Tadeu has published expressions for 2.5D Green’s functions in the ground. In subsequent papers, he has adapted this approach for vibro-acoustic problems through a BEM analysis in the case of an oblique acoustic wave impinging on an infinite structure [9,10].

The objective of the present paper is to analyse ground-structure interaction for point-like or incoherent line excitations. In section 2, the 2.5D formalism of the Green’s functions is briefly re-called and its implementation in a 2D BEM code is presented. In section 3, a numerical validation is considered. In section 4, examples are reported and discussed, including comparisons with 2D calculations.

2. 2.5D BEM FORMALISM
2.1. Green’s functions
The 2D FEM/BEM program MEFISSTO [2] is based on a combined use of FEM and BEM. In practice, a given problem is subdivided into sub-domains, usually characterised by media having different physical properties. Each sub-domain can be modelled either by FEM or BEM, the latter being necessary for unbounded domains. FEM matrices are condensed, corresponding to the outer nodes of a given sub-domain, thus leading to a full matrix analogous to a BEM matrix. The sub-matrices are matched
by means of the classical expressions for continuity of displacement and/or stress at the common boundaries. The approach is described in detail in [2]. Transforming MEFISSTO into a 2.5D program involves both the modification of its FEM and BEM parts. In this work, only the transformation of the BEM part is undertaken. As we shall see it is mostly based on the use of a special Green's function, whereas the 2.5D transformation of the FEM part would involve the development of finite elements as proposed by Finnvenden [12]. This restriction limits the applicability of the approach to isotropic homogeneous sub-domains.

The general 2.5D BEM solution to point excitations in infinite media, both for airborne or structure-borne/ground problems, is based on the study of a set of 2D problems. Each 2D problem corresponds to a line excitation with a given wavenumber dependency \( k_z \) along the infinite \( z \) direction. The solution to a point excitation (acoustic source or force) is then simply obtained by a Fourier integration over the individual solutions.

Consequently the 2.5D BEM approach relies upon the knowledge of a set of 2.5D Green's functions. Each function is the solution to an excitation \( P_2 \), which consists of a Dirac function in the 2D \( x \)-\( y \) plane and a sinusoidal dependency along \( z \):

\[
P_2 = \delta(x)\delta(y)e^{i(k_z z - \omega t)}
\]

Tadeu begins from the expression of the 3D Green's solution for a point load \( P_3 = \delta(x)\delta(y)\delta(z) \) as a combination of two potentials \( A_p \) and \( A_s \), which are solutions of two Helmholtz equations [9]. He uses the fact that the response to load \( P_2 \) can be obtained by applying a spatial Fourier transform in the \( z \)-direction, to these Helmholtz equations. Using the properties of Fourier transforms and the expressions for Green's tensors of displacement \( G \) and stresses \( T \) in terms of \( A_p \) and \( A_s \) leads to the 2.5D expressions of \( G \) and \( T \). These expressions are fully described in [9].

When programming Tadeu's expressions for \( G \) and \( T \), a number of verifications can be made to ascertain their correctness. First, the values for \( k_z = 0 \), must correspond to the 2D solution. Second, the well known 3D Green’s 3\( \times \)3 tensors must be recovered if the Fourier integration over \( k_z \) is made:

\[
\bar{G}(x, y, z) = \int_{-\infty}^{\infty} G(x, y, k_z) e^{-ik_z z} dk_z
\]  

The double bar denotes a 3\( \times \)3 tensor; \( G_{ij} \) is the displacement in the \( i^{th} \) direction at point \( x_M, y_M, z_M \) for a unit force in the \( j^{th} \) direction at point \( x_Q, y_Q, z_Q \); \( x, y, z \) are the relative components between \( Q \) and \( M \); \( dk_z \) is the resolution of the \( k_z \) sampling. When evaluating (1) the aliasing limit of \( z_{\text{max}} = \pi/dk_z \) must be respected.

2.2. BEM approach

Figure 1 shows a typical 2D situation. In this case, three sub-domains with different properties must be considered: the half infinite ground, a building and a tramway platform, both infinite along \( z \). This problem is solved for a vertical, infinitely long line
excitation, which appears as a vertical point excitation in the \(xy\) plane with a \(k_z\) dependency along \(z\). It corresponds to a wave impinging on the structure with an increasing angle as \(k_z\) increases. \(k_z=0\) corresponds to the 2D solution with cylindrical waves impinging normally on the building.

For each sub-domain the 3D displacement \(\vec{u}\) can be written [17] as

\[
\vec{u}(M)\vec{u}(M) = \int_S [\vec{G}(Q,M)\vec{u}(Q) - \vec{G}(Q,M)\vec{T}(Q)]dS + \vec{h}(E,M)
\]

(2)

where \(\vec{T}\) represents the stresses; \(\vec{h}\) is the free field excitation; \(\vec{G}\) and \(\vec{T}\) are the 3\(\times\)3 Green’s tensors, respectively, for displacement and stresses, as given in [9]; \(\vec{G}(M)\) is a tensor which is related to the position of \(M\) with regard to the sub-domain and is unity for points away from \(S\). For the half-infinite medium, \(S\) is first considered to be the portion of the problem contained in a finite half cylinder or radius \(R\) (see Figure 1). By letting \(R\) tend towards infinity and by using the radiation properties of \(G\) and \(T\), only the upper free surface remains. Free traction conditions on the ground-air interface are assumed. Even then, the top ground-air surface, which remains, is of infinite extent. Proper radiation considerations can be omitted in practice if one meshes the top surface over a length of a few Rayleigh wavelengths around the region of interest since global losses are usually very important.

Although \(\vec{u}, \vec{T}, \vec{G}\) and \(\vec{T}\) are 3D vectors and tensors, the major advantage of the present approach is that only contour lines in the \(xy\) plane need to be discretised. Once the programming effort has been made, the computation for a given \(k_z\) is only slightly more expensive than for the corresponding 2D situation and the increase of computation time is solely due to the addition of the \(u_z\) and/or \(t_z\) components at each node. The numerical implementation of equation (2) is straightforward [17] and consists of
discretising the boundaries in a finite number of meshes and nodes. Constant, linear
or quadratic elements can be programmed. Here, only linear elements have been
implemented. By considering equation (1) at each node and in each sub-domain, one
obtains a square, full matrix system in terms of the unknown displacement and stress
components at the nodes.

The major difficulty, as for any BEM approach, is the treatment of the singular
terms, which arise when the distance between $M$ and $Q$ in equation (1) tends to zero. By
considering the behaviour of each component of $\vec{G}$ and $\vec{F}$ when $r$ tends to zero, one
can express the limit behaviour in an analytical form in terms of $\log(r)$ or $1/\sqrt{r}$
functions which can be analytically computed. In practice, these singular terms are
subtracted from $\vec{G}$ and $\vec{F}$ so that numerical integration schemes can be employed. The
analytical integration is added to the regularized integrals. Once the solution is obtained
for a chosen set of $k_z$ wavenumbers, the values of $\vec{u}$ (or $\vec{f}$) can be computed for any
point force using equation (1).

2.3. Validation
In order to validate the 2.5D program, a simple case initially was considered. The
approach was somewhat analogous to the image source approach used in acoustics. In
case 1, consider a unit force parallel to an infinite rigid top surface of semi-infinite
ground. Case 2 is obtained by removing the free surface and by introducing a mirror
excitation at the symmetrical position with respect to the surface. The 2D computations

![Figure 2](image.png)

**Figure 2.** Computation of the $\vec{G}, \vec{f}$ term at $f=100$ Hz. Horizontal component of the
velocity, computed by (....................) summing the direct plus the image
source 3D solutions and (__________) 2.5D BEM computation.
show that the horizontal displacement, in both computations, are nearly identical. In 3D, two calculations are made.

For the second case, the solution can be directly computed by summing the two 3D free field Green’s functions [17] for the real and mirror sources. The 2.5D solution to the first case is obtained using MEFISSTO-2.5D. The ground here considered is made of loess with characteristics \( E=233 \) MN/m\(^2\), \( \rho=2400 \) kg/m\(^3\), Poisson’s coefficient \( \nu=0.24 \) and a loss factor \( \eta=0.1 \); the unit horizontal force is placed 5 m below the rigid boundary.

Figure 2 shows at 100 Hz, the horizontal velocity along the vertical axis between the surface and the unit force, as a function of the distance. Both solutions are found to be very similar. It must be noted that this test only validates the \( \overline{G}f \) terms in equation (2) since \( \overline{u} = 0 \) on the surface.

A further comparison was made with the results of a computer program, CASC-S [13], for layered infinite media which is based on a transfer matrix semi-analytical approach. Figure 3 represents the two situations considered. The analytical computation considers the case of an infinite 200 mm concrete wall in contact, on one side, with ground made of loess \( (E=269 \) MN/m\(^2\), \( \rho=1550 \) kg/m\(^3\), \( \nu=0.257 \) and \( \eta=0.10 \)) and
excited by sets of uncorrelated P- or SV-waves arriving from all directions in the ground. The average velocity on the wall is normalised with respect to the velocity at the same position in an infinite ground when excited in the same manner. The BEM computation has been carried in the case of a vertical 200 mm foundation of infinite length, in the z-direction, but with a finite height of 5 m. The excitation is either an infinite line of coherent vertical forces (2D), a point force or an infinite incoherent line of vertical forces (2.5D). In Figure 4 is shown the average velocity–bending and longitudinal-components on the structure normalised with respect to the velocity in the half space at the same location and for the same kinds of excitations. The values therefore are of insertion loss. The insertion losses for both components of the velocity are compared with the analytical insertion losses for P- and SV-waves, respectively.

Although the analytical and BEM problems are not strictly identical it seemed beforehand that the horizontal component of velocity on the foundation when excited by an infinite incoherent line source could be viewed as being similar to the horizontal velocity component on infinite ground when excited by a diffuse set of P-waves. Similarly the vertical-longitudinal component of the velocity obtained by BEM has been compared with the vertical component obtained in the analytical case for an incident SV diffuse field.

Figure 4 shows a good agreement between the results of the analytical approach and of the BEM calculation for incoherent line or point sources. The 2D BEM solution for longitudinal response to a coherent line source shows a greater discrepancy with the 2.5D results than for the bending wave results. This justifies the need for a 2.5D calculation. As already mentioned, the two problems are not strictly identical but the similarity of the results shows that the analytical approach will lead to very satisfying results for this simple case. These comparisons also can be seen as a further validation

Figure 4. Computation of the velocity on a vertical foundation in contact with the ground on one side relative to the velocity in full ground. ............ Semi-analytical ........... 2D line ○ point □ 3D line.
of the 2.5D BEM program, which is capable of handling more complex problems than the analytical approach.

3. NUMERICAL APPLICATIONS
3.1. Isolating layers on foundations
MEFISSTO was originally developed in order to study the isolation of foundations by means of layers of soft material such as polystyrene [2,5,16]. Numerical results of 2.5D computations can be found in [6] and measurements reported in [4] for the configuration represented by Figure 5. It consists of a trough like structure made of five 200 mm concrete walls and base. The walls are 3.5 m high, with 2.5 m under ground level. The ground surface was excited by dropping heavy masses on 1x1 m² concrete

![Figure 5. Plan of experimental facility for the measurement of the efficiency of isolating layers and schematic 2D/2.5D representation.](image-url)
slabs regularly spaced along two lines parallel to the structure (see Figure 5). The masses and dropping heights were selected to induce ground vibration spectra similar to those generated by tramways. This is demonstrated in Figure 6 where the vertical surface velocities measured 3 m away from a mass dropped on two of the slabs are compared with the surface velocity measured 3 m from a tramway not far away from the test site. The results are similar.

Velocity levels on the walls were averaged for several measuring points. They also were averaged for impacts at seven locations along a given line (four positions in practice due to symmetry) thus reproducing the incoherent excitation due to the passing of a tramway. Two sets of measurements were conducted; first, without protection and second, with 100 mm of polystyrene placed between the ground and the vertical wall facing the excitation line. The efficiency of the protective layer has been defined as the difference of the averaged velocity levels on the treated wall without and with protection. The ground characteristics were obtained according to [15] where $E = 44.4$ MN/m$^2$, $\rho = 1500$ kg/m$^3$, $\nu = 0.25$, $\eta = 0.08$.

Figure 7 shows the measured efficiencies and computed values using 2D BEM, assuming infinite coherent line excitation. Measured results show a reduction of wall vibrations between 16 and 63 Hz with an efficiency of the order 5-10 dB. Above 63 Hz, the measured efficiency is lower and lends to 0 dB. On the other end, the computed values are of the order of 10-15 dB. The dip at 50 Hz corresponds to the mass-spring frequency of the layer-wait combination. Above this frequency the predicted efficiency is again positive as would be the case for an ideal mass-spring system. Several explanations have been proposed for this discrepancy at high frequencies. The ground

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Figure 6. Measured velocity spectra 3 m from the excitation. ⊗ dropped masses at two locations; ---- tramway excitation.
is not likely to behave as a simple homogeneous dry half space. There may be differential movement between the ground and structure. The characteristics of the resilient layer may be affected by the ground loading.

Can the problem be approached using a 2D model? This question was considered particularly worthy of investigation. At this stage it must be pointed out that the structure measured had a finite length of 10 m and therefore could not be considered as infinite, as required by the 2.5D approach. In order to assess the effect of this finite length, 2D computations were undertaken for a fully buried 200 mm finite-length wall, which forms part of a rectangular tunnel. The wall was considered without treatment and then when covered by 100 mm of polystyrene. Different excitations were considered.

It was observed that the efficiency of the protective layer on the bending velocity was little modified by increasing the length from 10 to 20 m. The spectra, although not strictly identical, show similar orders of magnitude. This suggests that the finite size of the testing facility is not of prime importance. Figure 8 shows, at 20, 31 and 50 Hz the vertical velocity $V_y$ along the line A-B-C-D-E (see Figure 5) as a function of $k_z$. A-B corresponds to the free surface between the line source and the structure. Along BC and DE, $V_y$ corresponds to longitudinal waves in the structure whereas along CD it can be associated with bending behaviour. On each graph the value of the Rayleigh wave number $k_R(f)$ is indicated and can be seen to be a limiting value above which $V_y(k_z)$

![Figure 7. Measured and 2D BEM computed values of isolation efficiency of 100 mm of elastified polystyrene. Velocity level without treatment minus velocity level with treatment. Measurement ---------- 2D FEM/BEM.](image)
rapidly decreases since a given $k_z$ corresponds to a wave impinging on the foundation with an angle equal to $\sin^{-1}\left(\frac{k_z}{k_R}\right)$.

The 2.5D solution for a point excitation is possible by extending the integration in term of $k_z$ in equation (1) up to infinity. In practice only a finite interval is allowed. Figure 8, shows that the required integration interval depends on the distance between source and receiver, the closer the receiver and the larger the integration range required.

In order to obtain the solution for an incoherent line excitation, the solution must be integrated for a set of incoherent point sources along the line. Figure 9, shows the 3D approach provided velocity for a vertical unit point force at $z = 0$, for an untreated structure. The horizontal axis follows the ABCDE contour, whereas the vertical axis corresponds to the $z$-axis parallel to the infinite structure. The maps show the attenuation, along the $z$-axis, is more rapid for increasing frequencies. The computation for an infinite incoherent source line is approximated by integration of a finite length; the higher the frequency the shorter the integration length required. Figure 10 confirms this, by showing the effect of increasing the integration length. The horizontal and vertical velocities along ABCDE are shown, at 50 Hz, relative to a computation made for 200 m.

Figure 11 represents, at 40 Hz and 160 Hz, the insertion loss on the vertical component of the velocity $V_y$ along ABCDE and the $z$-direction for a unit vertical force at $z = 0$, when 100 mm of polystyrene is added along the front vertical wall (BC). The attenuation can clearly be seen on the foundation contour BCDE. Larger reductions are obtained at 160 Hz.

Figure 12 shows, along BC (the treated wall), the attenuation of velocity at different frequencies, for both a coherent and an incoherent infinite line source. The differences
Figure 9. Representation of the 3D vertical velocity $V_y$ along A-B-C-D-E and $z$, after integration, in terms of $k_z$. Excitation at $x=A$, $z=0$ co-ordinates according to Figure 5. Line AB is the ground/air interface, BCDE is the buried structure contour.

Figure 10. Effect of length, along the $z$-axis, of an incoherent line source at 50 Hz. Velocity level along the ABCDE contour, relative to the velocity of an infinite incoherent line. Line AB is the ground/air interface, BCDE is the buried structure contour $L=+4$ m, * 10 m, □ 20 m, □ 40 m, ▲ 80 m.
Figure 11. Insertion loss for the vertical velocity \( V_y \), in terms of \( k_z \). Excitation at \( x=A, z=0 \); 100 mm of polystyrene placed along a buried wall (segment BC, see Figure 5). Line AB is the ground/air interface, BCDE is the buried structure contour.

Figure 12. Velocity attenuation along a treated buried wall (line BC in Figure 5) due to the addition of an isolating layer. Vertical line excitation. Upper graphs: horizontal attenuation, lower graphs: vertical attenuation. ______ 2D (coherent) __________ 3D (incoherent).
between both types of excitations are more clearly seen for the horizontal (bending) component. The coherent source computation shows stronger variations along the wall for this component, the incoherent results for bending being smoother.

Figure 13 represents the rms velocity levels, without and with treatment, for coherent and incoherent line sources. The 3D predicted velocity levels are lower than the corresponding 2D values. The differences between both types of sources are reduced for increasing frequencies.

Figure 14 shows the efficiency of a 100 mm isolating layer for infinite coherent lines of excitation and 20 m- or 200 m-long incoherent lines. The consideration of 3D sources results in a reduction of the efficiency, especially for the bending component. However, the efficiency remains largely positive over the whole spectrum. The consideration of a more realistic source of excitation does not suffice to explain the discrepancy between computation and measurements indicated in Figure 7. Either some unidentified measurement problems or a discrepancy between real and simulated situations must be held responsible. More carefully controlled measurements/computations are still needed before using the BEM approach as a simulation tool. It is clear that the real case is more complicated than the idealised models used for prediction. Although not reported, computations with more complex descriptions have not, so far, removed or reduced the discrepancy between measurement and prediction.

Figure 13. 2D (coherent line source) and 3D (incoherent line source) predicted vertical velocity levels of a buried wall (line BC in Figure 5), without and with a protective layer. ________ 2D _________ 3D, ○ with polystyrene layer.
Figure 14. 2D and 3D predicted efficiency of a 10 cm polystyrene layer placed along a buried wall (line BC in figure 5). + 2D, * 3D (20 m incoherent line source), ○ 3D (200 m incoherent line source).

Figure 15. Velocity level difference between surface vertical velocity level (LV1) and foundation 'bending' velocity level. Influence of the excitation type: _______ 2D, * 3D point, ○ 3D 4 m, □ 3D 10 m, ∆ 3D 20 m.
3.2. Surface-foundation transfer functions

Another important potential application for predictive models is their ability to estimate the transmission of vibrations into foundations from surface velocity measurements. A knowledge of surface induced vibrations and the computation of velocity transfer functions between surface and foundations is particularly important for building and tramway design and construction. Similar transfer functions were calculated in Figure 4 and the comparison between 2.5D BEM and a semi-analytical approach appears satisfactory which arguably validates both models. Measurements, not reported, were also found to be in poor agreement with calculations. Again the 3D effect was not sufficient to approximate the experimental results.

Figure 15, shows the different surface/foundation transfer functions computed for different source assumptions. The largest difference, of the order of 10 dB, occurs between 2D (coherent vertical line force) and vertical point force excitation. So, again 3D effects are found to be significant but not sufficient to obtain a good agreement between computations and measurements.

4. CONCLUSIONS

The modelling of ground-structure interaction is a complex process. It can be modelled by using 2D BEM models, which assume that the problem can be described by simple homogeneous isotropic sub-domains. The source and the geometry are then assumed to be constant along an ‘infinite’ direction.

The combined use of FEM allows for a somewhat more complex medium, where sub-domain properties may vary between elements. The connection between sub-domains assumes a perfect continuity of displacements and stresses. However, these assumptions are seldom verified.

This paper has concentrated on the source assumption. As suggested by several authors [9-11], it is possible to adopt a 2.5D approach where the geometry remains 2D but where sources are 3D, such as point forces or incoherent lines. The trick is to solve a set of modified 2D problems and then to Fourier transform the solutions. By using modified Green’s functions, each 2D problem still has the advantage of considering 2D boundaries, but each variable (stresses and displacements) is now 3D. Validation has been partially obtained in simple cases.

An important conclusion of this work is that consideration of 3D sources is easily feasible starting from an existing 2D BEM program and that the consideration of incoherent sources leads to significantly different results.

A further extension of this work will be the introduction of 2.5D FEM elements which are the counterpart of the special Green’s functions employed here, since a wave description along the infinite direction has already been proposed by Finnvenden [12] under the name of spectral finite elements. Other aspects such as imperfect ground-structure interaction should also be considered.

REFERENCES


